Feeding the World

I. UNIT OVERVIEW & PURPOSE:

There are concerns about protein supplies for the world population and demand outgrowing supply within the next fifty years. This unit has two lessons where the student will analyze data regarding protein supply and population growth.
Students are going to be guided through the data using provided handouts and will be using exponential regressions and growth equations to predict future populations and protein consumption.

II. UNIT AUTHOR:

Monica Tomasik, Thomas Dale High School, and Chesterfield County Public Schools.

III. COURSE:

Mathematical Modeling: Capstone Course

IV. CONTENT STRANDS:

Number and Operations, Algebra, and Data Analysis

V. OBJECTIVES:

The student will do the following:

1. Use technology to find a regression model for given data.
2. Predict future populations using a regression.
3. Discuss factors that affect population growth.
4. Use $A = Pe^{rt}$ to predict world populations.
5. Find how much protein average humans eat in a year and compare that to protein production
6. Predict when the population will outgrow our protein supply.

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MEP.1 Solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.

MEP.2 Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

MEP.12 Transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Select and use appropriate representations for analysis, interpretation, and prediction.

MEP.14 Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. Use a transformational approach to graphing. Use graphing calculators as a tool to investigate the shapes and behaviors of these functions.

VII. CONTENT:

The students will use the data provided to explore world population and protein consumption/production. They will then explore the different factors affecting

population growth and the fact that at some point in the near future, there will possibly be a protein shortage. They will be asked to brainstorm different solutions to this problem.

VIII. REFERENCE/RESOURCE MATERIALS:

This unit will require the use of TI-nSpire calculators and the provided handouts.

IX. PRIMARY ASSESSMENT STRATEGIES:

Each lesson has a handout which guides the student through the lesson. There are practice problems and a short 2-3 question assessment for each.

X. EVALUATION CRITERIA:

Each assessment has an answer key which the instructor can use to mark answers right or wrong. It is left to teacher discretion as to how the questions and quizzes will count

XI. INSTRUCTIONAL TIME:

This Unit will require 2 full classes of at least 50 minutes each.

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in collaboration with
Mr. Michael Bolling, Virginia Department of Education

U.S. Population

The table to the right represents all of the U.S. Census Data recorded in the history of the country. The census is taken every 10 years. We will be letting 1790 be year 0 since it was the first year the census was ever recorded. Each decade after that would increase by 1. Use this data to make a table in your TI-nSpire and then graph and find an appropriate regression.

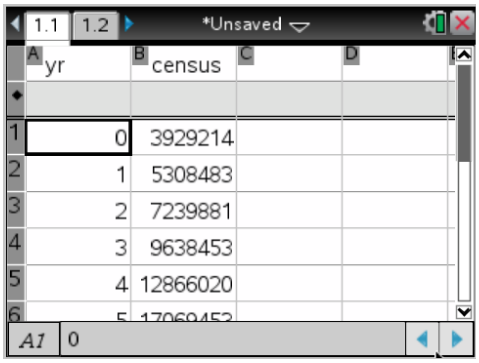
All data is from the United States Census Bureau.

The census is taken every ten years. Let 1790 be represented by 0, therefore, 1800 would be 1.

Year	U.S. Population
0	3,929,214
1	5,308,483
2	7,239,881
3	9,638,453
4	12,866,020
5	17,069,453
6	23,191,875
7	31,443,321
8	39,818,449
9	50,189,209
10	62,947,714
11	76,212,168
12	92,228,496
13	106,021,537
14	122,775,045
15	131,669,275
16	150,697,361
17	179,323,175
18	203,184,772
19	226,545,805
20	248,709,873
21	281,421,906
22	308,745,538

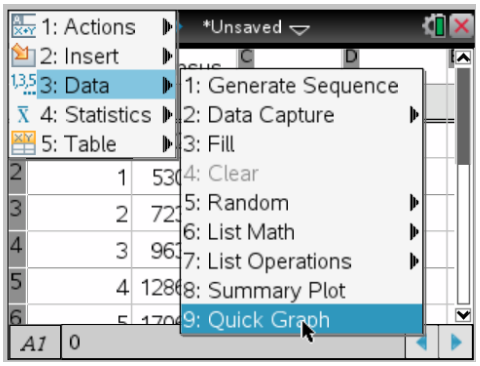
Step 1

Open a new document and choose List & Spreadsheet, then fill in the data. *See the picture to the right.



Step 2

Once the data are entered and you have checked it for any errors, hit the menu button on the calculator and then choose



and Quick Graph.

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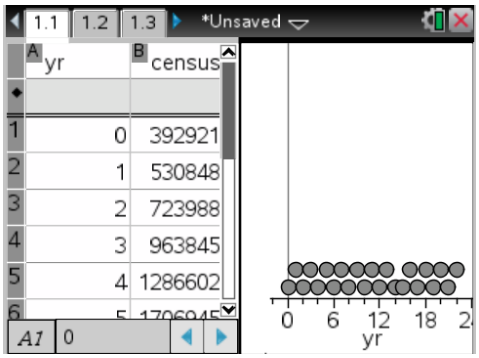
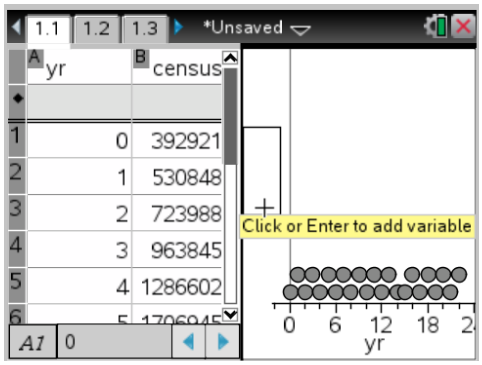
Step3

Your calculator screen should now look like this:



You will need to format your graph by moving your cursor along the y-axis until a rectangle appears like the picture below: Once you click on the rectangle, you will want to pick census.

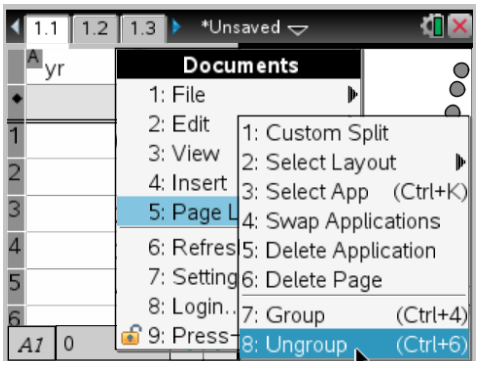


Free Response

1. What parent function does the graph look like? _____

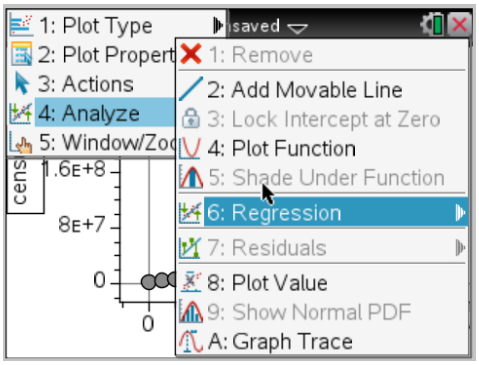
Step 4

To get a better view of your graph, let's put it on its one page. Use the Doc button on your calculator and select Page Layout and Ungroup.



Step 5

Find the equation of the curve of best fit. While viewing the graph, click the Menu button and choose Analyze, Regression and then pick the regression function that you think best fits the data.



Free Response

2. What is the equation of the curve of best fit? _____

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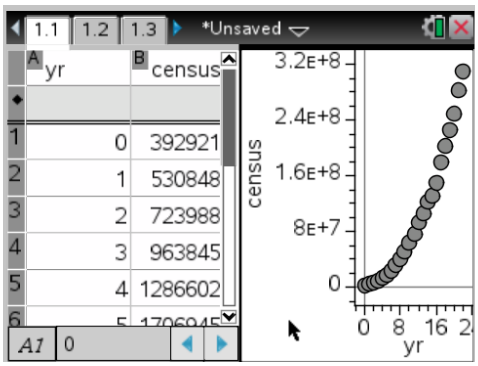
3. Predict the U.S. population in the year 2020 _____

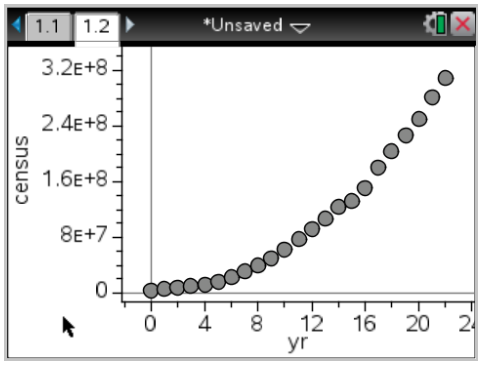
   and in the year 2050 _____

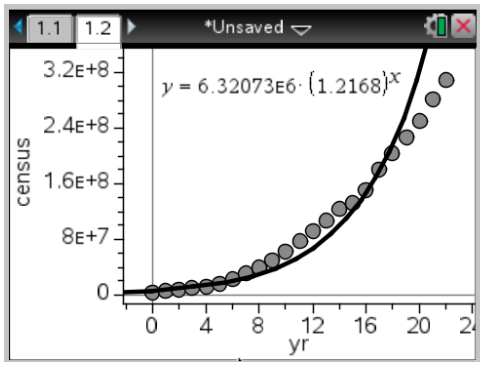
4. Does the data on your graph match the regression curve exactly? _____

   If no, what could be causing the population to not follow the regression curve?

Answers:







1. Exponential Function
2. $y = 6.3207\times10^{6} \cdot (1.2168)^{x}$
3. $x = 23 \quad y = 57{,}651{,}000$ and $x = 26 \quad y = 1{,}038{,}640{,}000$
4. No, the actual population growth slowed down. Answers may vary, population growth has slowed for many reasons, immigration has slowed, couples are having fewer children, etc.

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U.S. Population

The table to the right represents all of the U.S. Census Data recorded in the history of the country. The census is taken every 10 years. We will be letting 1790 be year 0 since it was the first year the census was ever recorded. Each decade after that would increase by 1. Use this data to make a table in your TI-nSpire and then graph and find an appropriate regression.

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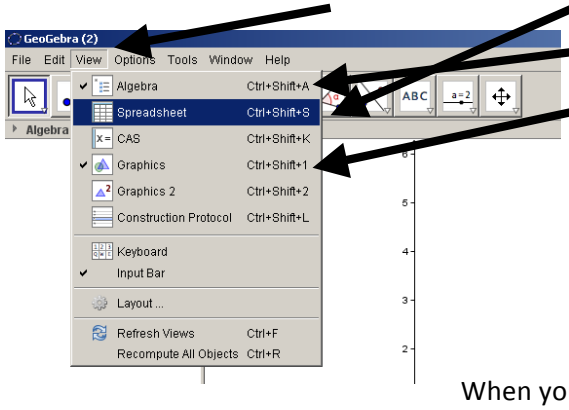
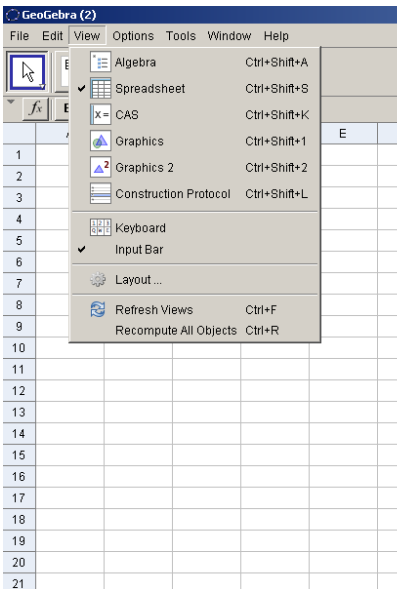
17	179,323,175
18	203,184,772
19	226,545,805
20	248,709,873
21	281,421,906
22	308,745,538

Step 1

Open GeoGebra. Click on "View" then select "Spreadsheet" and de-select "Algebra" and "Graphics".



When you are done, your screen should look like this →



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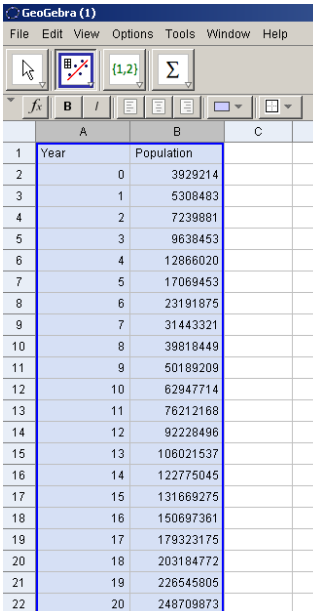
Step 2

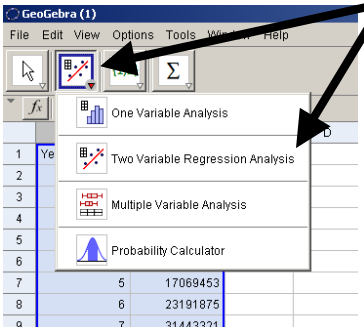
Label column A as “Year” and column B as “Population”, then enter your data.



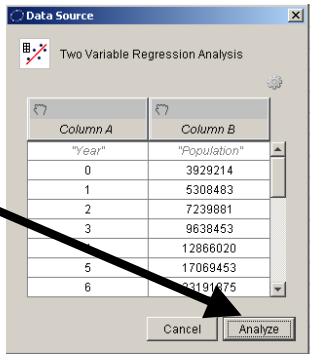
Step3

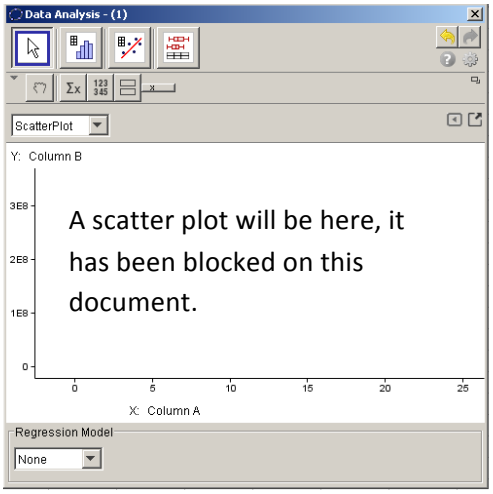
Highlight your data (including the titles) and then select the second tool button.

Select “Two Variable Regression Analysis.”



A box will appear on your screen; Click the Analyze button





The screen to the left should appear.

Free Response

1. What parent function does the graph look like? _____

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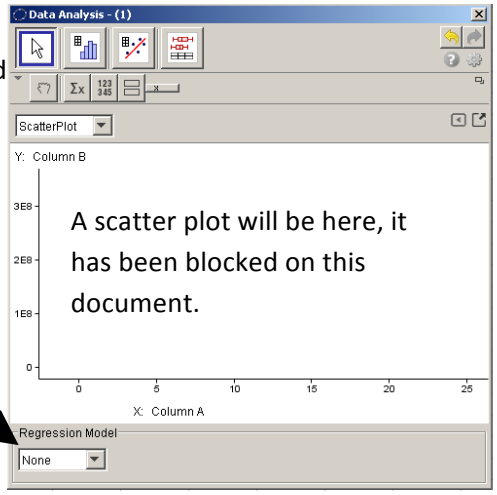
Step 4

To find a regression model, click on the drop down box at the bottom left and chose the model corresponding to your answer in the first free response question.



Free Response

2. What is the equation of the curve of best fit? _____

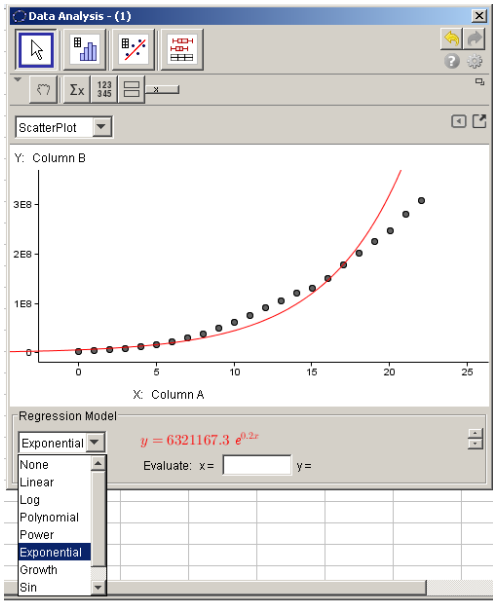
3. Predict the U.S. population in the year 2020 _____

   and in the year 2050 _____

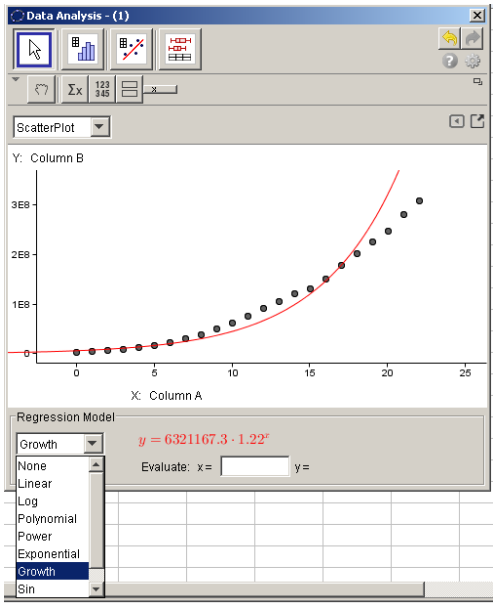
4. Does the data on your graph match the regression curve exactly? _____

   If no, what could be causing the population to not follow the regression curve?

Answers:



Or



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1. Exponential Function or Growth Function
2. $y = 6321167.3e^{0.2x}$ or $y = 6321167.3 \cdot 1.22^{x}$
3. $x = 23$ $y = 628{,}857{,}000$ or $y = 612{,}454{,}000$ and $x = 26$ $y = 1{,}145{,}850{,}000$ or $y = 1{,}112{,}120{,}000$
4. No, the actual population growth slowed down. Answers may vary, population growth has slowed for many reasons, immigration has slowed, couples are having fewer children, etc.

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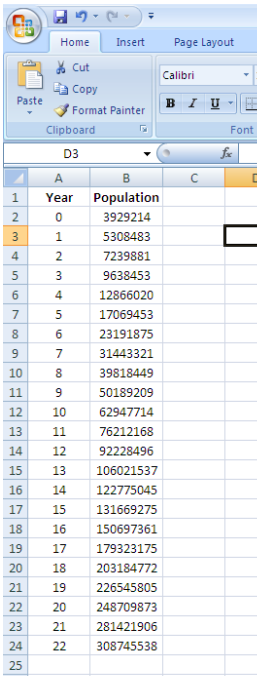
17	179,323,175
18	203,184,772
19	226,545,805
20	248,709,873
21	281,421,906
22	308,745,538

Step 1

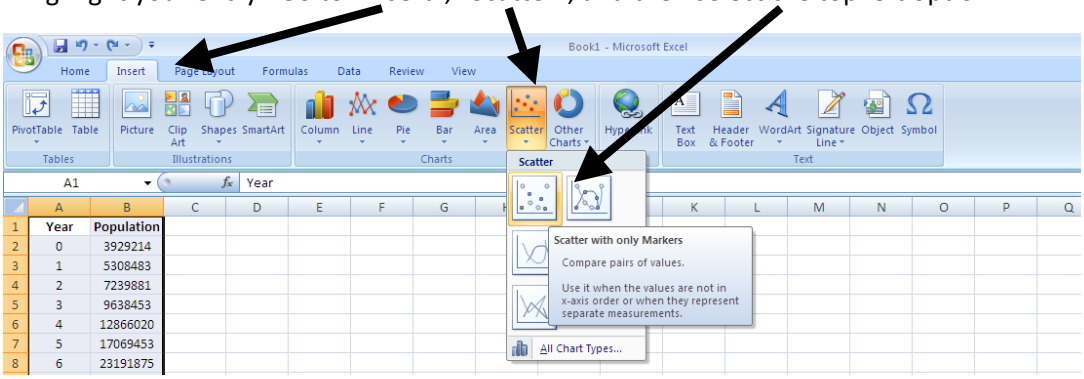
Open an Excel worksheet. Enter your data labeling column A as "Year" and column B as "Population".



	A	B
1	Year	Population
2	0	3929214
3	1	5308483
4	2	7239881
5	3	9638453
6	4	12866020
7	5	17069453
8	6	23191875
9	7	31443321
10	8	39818449
11	9	50189209
12	10	62947714
13	11	76212168
14	12	92228496
15	13	106021537
16	14	122775045
17	15	131669275
18	16	150697361
19	17	179323175
20	18	203184772
21	19	226545805
22	20	248709873
23	21	281421906
24	22	308745538

Step 2

Highlight your entry. Go to "Insert", "Scatter", and then select the top left option.



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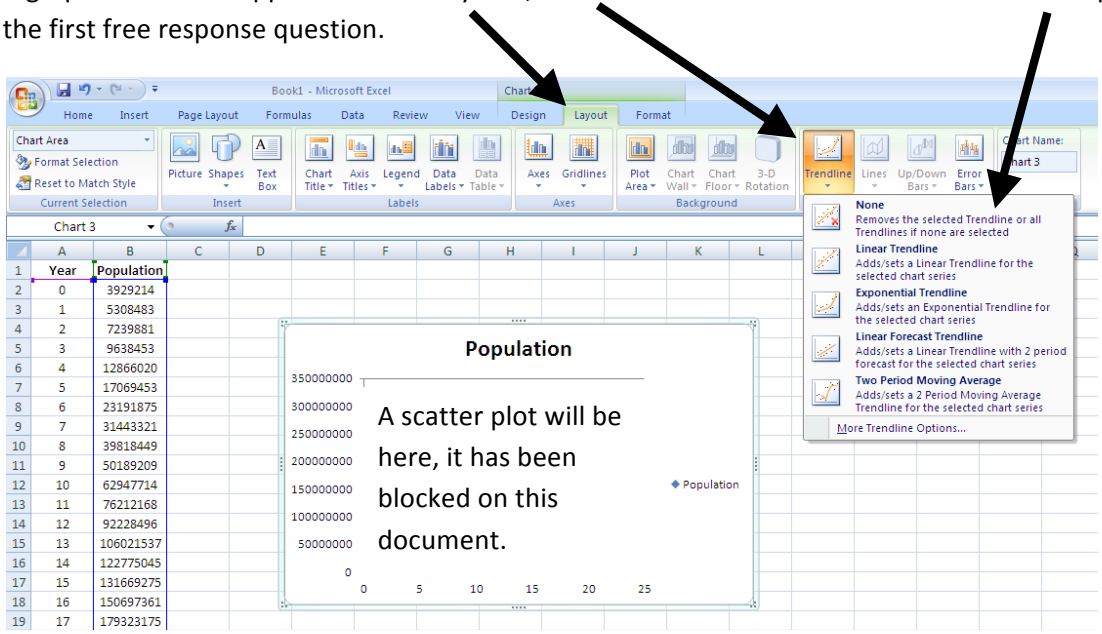
Free Response

1. What parent function does the graph look like? _____

Step3

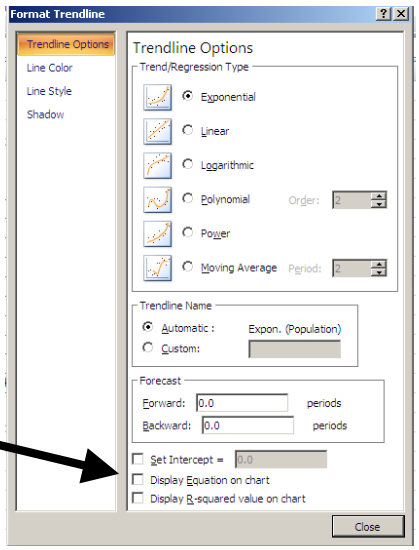
A graph box should appear. Select "Layout", "Trendline" and then click the Trendline corresponding to your answer in the first free response question.



Step 4

Right click on the trendline and the screen to the right should appear.

Select the box for "Display Equation on chart"



*You may have to click and drag the equation in order to see it better.

Free Response

2. What is the equation of the curve of best fit? _____

3. Predict the U.S. population in the year 2020 _____

   and in the year 2050 _____

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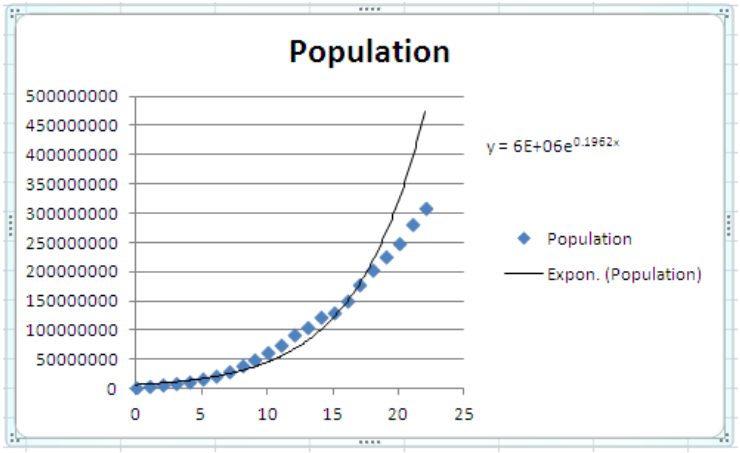
4. Does the data on your graph match the regression curve exactly? _____

   If no, what could be causing the population to not follow the regression curve?

Answers:



1. Exponential Function
2. $y = 6{\times}10^6 e^{0.1952x}$
3. $x = 23$ $y = 534{,}515{,}000$ and $x = 26$ $y = 960{,}025{,}000$
4. No, the actual population growth slowed down. Answers may vary, population growth has slowed for many reasons, immigration has slowed, couples are having fewer children, etc.

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Curves of Best Fit

Using the given data, find the equation for the curve of best fit.

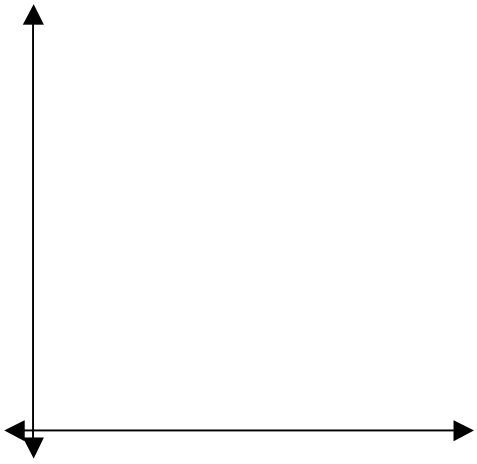
1.

Global Estimates of Cumulative HIV/AIDS Cases, 1980-96

Year	HIV Infection (million)	AIDS Cases (million)
1980	0.2	0
1981	0.6	0
1982	1.1	0
1983	1.8	0.1
1984	2.7	0.2
1985	3.9	0.4
1986	5.3	0.7
1987	6.9	1.1
1988	8.7	1.6
1989	10.7	2.3
1990	13.0	3.2
1991	15.5	4.2
1992	18.5	5.5
1993	21.9	6.9
1994	25.9	8.5
1995	30.6	10.4
1996	36.2	12.5

Compiled by Worldwatch Institute from: Global AIDS Policy Coalition, Harvard School of Public Health

Draw a sketch of the graph



Which parent graph does this graph most resemble?_____

What is the equation of the curve of best fit?

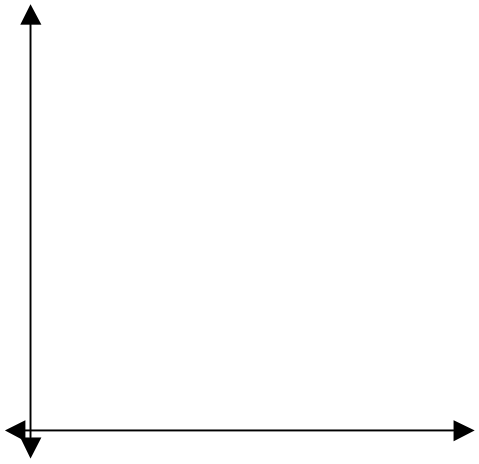
_____

2.

Worldwide Carbon Emissions Dataset

Year	Carbon Emissions (mill. tons)	Year	Carbon Emissions (mill. tons)
1950	1620	1980	5170
1955	2020	1981	4998
1960	2543	1982	4959
1965	3095	1983	4945
1966	3251	1984	5114
1967	3355	1985	5286
1968	3526	1986	5472
1969	3735	1987	5593
1970	4006	1988	5809
1971	4151	1989	5914
1972	4314	1990	5943
1973	4546	1991	6010
1974	4553	1992	5926
1975	4527	1993	5919
1976	4786	1994	5989
1977	4920	1995	6080
1978	4960		
1979	5239		

Draw a sketch of the graph



Which parent graph does this graph most resemble?_____

What is the equation of the curve of best fit?

_____

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Curves of Best Fit (Answer Key)

Using the given data, find the equation for the curve of best fit.

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1983	1.8	0.1
1984	2.7	0.2
1985	3.9	0.4
1986	5.3	0.7
1987	6.9	1.1
1988	8.7	1.6
1989	10.7	2.3
1990	13.0	3.2
1991	15.5	4.2
1992	18.5	5.5
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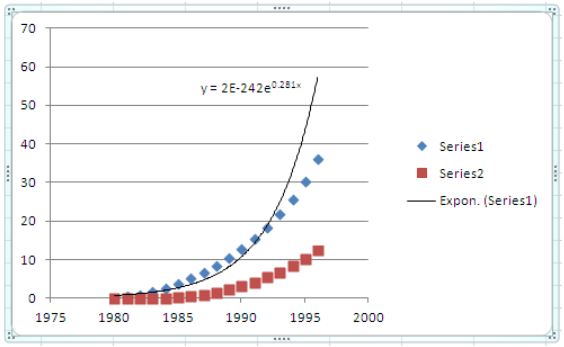
1994	25.9	8.5
1995	30.6	10.4
1996	36.2	12.5

Compiled by Worldwatch Institute from: Global AIDS Policy Coalition, Harvard School of Public Health

Draw a sketch of the graph



Which parent graph does this graph most resemble?___Exponential Function_____

What is the equation of the curve of best fit?

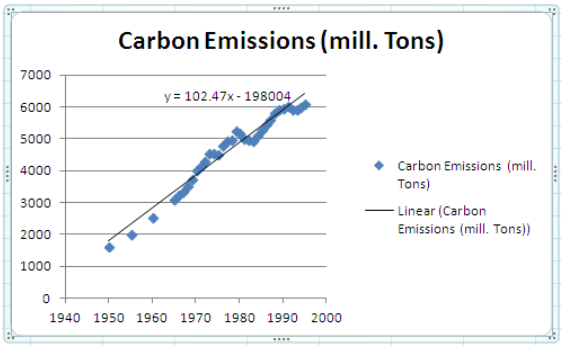
_____$y = 2\times10^{242}e^{0.281x}$_____

2.

Worldwide Carbon Emissions Dataset

Year	Carbon Emissions (mill. tons)	Year	Carbon Emissions (mill. tons)
1950	1620	1980	5170
1955	2020	1981	4998
1960	2543	1982	4959
1965	3095	1983	4945
1966	3251	1984	5114
1967	3355	1985	5286
1968	3526	1986	5472
1969	3735	1987	5593
1970	4006	1988	5809
1971	4151	1989	5914
1972	4314	1990	5943
1973	4546	1991	6010
1974	4553	1992	5926
1975	4527	1993	5919
1976	4786	1994	5989
1977	4920	1995	6080
1978	4960		
1979	5239		

Draw a sketch of the graph



Which parent graph does this graph most resemble?__Linear Function_____

What is the equation of the curve of best fit?

_____y=102.47x-198004_____

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Feeding the World

World Population

In our last lesson, we found an equation to model the U.S. population growth since 1790. Unfortunately, we also discovered that the equation did not match the population growth in the past 50 years. Population does follow an exponential model, but that model is forever changing because the rate at which people die and reproduce are changing as the world evolves. To find the growth of the world's population, we will need to use the *rate of natural increase* (r). The formula to find r is: Birth Rate (b) – Death Rate (d).

According to the data from the Population Reference Bureau, the current rate of natural increase is 1.2% and the world's population is 7.1 billion people.

Exponential Growth Model

To write equations for exponential growth, we use the formula $A = Pe^{rt}$. In this equation, A is the final population, P is the initial population, r is the rate of increase (rate of natural increase), and t is time in years. We can use this to predict the population in the future.

Write an equation modeling the world's population using $A = Pe^{rt}$ and the information from above.

1. What is our P? _____
2. What is our r? _____
3. What is the final equation? _____

Feeding the World

Protein is a crucial component of a human being's diet. According to an article from www.today.com, the average human needs to consume 0.4 grams of protein per pound per day.

4. If the average human adult weighs 136.7 pounds, how much protein will that person need to consume in one day?_____ in one year? _____ (Walpole, 2012)
5. If the average child weighs 60 lbs, how much protein will that child need to consume in one day?_____ in one year? _____
6. If 25% of the world's population is children (younger than 14), and the world's population is 7.1 billion people, how many children are currently in the world? _____ how many adults?_____
7. Using the information from #4 - #6, how much protein will the world consume in one year? _____

There are many forms of protein for human consumption; the following is a list of some of the major forms of protein consumed:

> Beef, Pork, Broiler (chicken), Copra, Cottonseed, Fish, Palm Kernel, Rapeseed, Peanut, Soybean and Sunflower Seeds

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According to a report from JP Morgan and the USDA, 245 tons of beef, pork and broiler and 268.16 tons of copra, cottonseed, fish, palm kernel, rapeseed, peanut, soybean, sunflower seeds were produced in 2012.

8. How much protein will we need to feed the world in 2020?_____
9. How much protein will we need to feed the world in 2050?_____

Free Response

1. Do you think that the rate of natural increase is getting larger or smaller as the world evolves? Why?_____
2. What are some alternative options for protein consumption in case the world cannot produce enough meat to satisfy the world's need for protein?

Answers

1. What is our P? __7.1 billion_____
2. What is our r? _1.2% = 0.012____
3. What is the final equation? __$A = 7.1e^{0.012t}$___
4. If the average human adult weighs 136.7 pounds, how much protein will that person need to consume in one day?__136.7*0.4 = 54.68grams__ in one year? __54.68*365 = 19958.2 grams (Walpole, 2012)
5. If the average child weighs 60 lbs, how much protein will that child need to consume in one day?_60*0.4=24grams____ in one year? __24*365=8760grams_____
6. If 25% of the world's population is children (younger than 14), and the world's population is 7.1 billion people, how many children are currently in the world? _7.1*.25=1,775,000,000___ how many adults?_____7.1-1.775=5,325,000,000_____
7. Using the information from #4 - #6, how much protein will the world consume in one year?
   _5,325,000,000*19958.2+1,775,000,000*8760=1.21826×10^{14}grams
   1 ton = 1000 kg and 1 kg = 1000 g so $\frac{121826000000000g}{1}\cdot\frac{1kg}{1000g}\cdot\frac{1ton}{1000kg}=121{,}826{,}000tons$

8. How much protein will we need to feed the world in 2020?_$7.1e^{0.012(7)}=7.72217$
   $7{,}722{,}170{,}000(.25)(8760)+7{,}722{,}170{,}000(.75)(19958.2)=132{,}502{,}000{,}000{,}000grams$
   $\frac{132502000000000g}{1}\cdot\frac{1kg}{1000g}\cdot\frac{1ton}{1000kg}=132{,}502{,}000tons$
9. How much protein will we need to feed the world in 2050?_$7.1e^{0.012(37)}=11.0684$
   $11{,}068{,}400{,}000(.25)(8760)+11{,}068{,}400{,}000(.75)(19958.2)=1.89919\times10^{14}grams$
   $\frac{1.89919\times10^{14}g}{1}\cdot\frac{1kg}{1000g}\cdot\frac{1ton}{1000kg}=1.89919\times10^{8}tons$

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Exploring U.S. Dietary Concerns

Research

Pick from the list of topics below. You will need to find data on this topic and then perform an analytical regression analysis to find any trends and make conclusions. The USDA and US Census are great resources.

- Beverage Consumption (soda, bottled water, etc)
- Snack Foods (type and/or quantity)
- Meat Consumption
- Dairy Consumption
- Obesity
- Diabetes

Analysis

Once you have researched your topic and gathered adequate data, plot your data and find a regression model using the methods learned in previous lessons.

Conclusions

Write at least three paragraphs about what you discovered and how you think it is affecting the US population (either negatively or positively). Make a poster depicting your topic and conclusion(s) and list three extension topics that could be further researched as a result of your conclusions.

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Analysis Rubric

Data Analysis	Accomplished No Deficiencies 10	Proficient Some areas need Development 5	Needs Improvement Considerable revision Needed 0
Data are presented to support the conclusions using the appropriate analyses	Data fully supports all conclusions	Some conclusions are not fully supported by the data	Insufficient data to supportstated conclusions
Suitable tables summarize data in a clear and meaningful way even to those unfamiliar with the information	Data are accurately presented in a suitable format, correctly titled and labeled, easy to interpret, and relevant to the purpose	Data are accurately presented but in need of some revision	Data not presented in a suitable format
Suitable graphs summarize data in a clear and meaningful way even to those unfamiliar with the information	Data are accurately presented in a suitable format, correctly titled and labeled, easy to interpret, and relevant to the purpose	Data are accurately presented but in need of some revision	Data not presented in a suitable format
Data reviewed and analyzed accurately and coherently	Data are appropriately evaluated using appropriate methods and valid conclusions are drawn	Data are evaluated and valid but incomplete conclusions are drawn	Data are not valid and/or incomplete or inaccurate conclusions are drawn
Proper use of descriptive and/or inferential mathematics	Data are analyzed in a valid manner consistent with the stated purpose of the	Data are analyzed but in an incomplete or inaccurate manner	Lack of examination

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